CONTAINER TRANSPORTATION NETWORK EQUILIBRIUM ANALYSIS
CONSIDERING DRAFT OF VESSEL

Syafi’i
Email : syafi@uns.ac.id

Abstract
This paper analyzes container transportation network equilibrium considering draft of vessels. Concept of load factor ($\lambda$) of ship is included in the model. Three players are considered, i.e. port administrator, ship companies (carriers), and shippers. Interaction of these players leads to Nash equilibrium problem. The result of the model calculation indicates that Hong Kong and Singapore port dominates container throughput in the world and the big vessel (3000 - 6000 TEU) is dominant in these ports. Conversely, the smaller port with depth less than 15 m dominated by 1000 TEU vessels. The result is inline with the reality. The other finding from the study is 6000 TEU vessels can enter port with depth less than 15 m such as port of Shanghai. Again, it is inline with reality. Validation of the model shows that coefficient of determination ($R^2$) is 0.95. It indicates the model provides good accuracy.

Keywords:
Container transportation, Nash Equilibrium, Network

INTRODUCTION
In the process of globalization, containerization is continuing to make a vital contribution to the rapidly growing international trade. It provides shippers with safe, easy and relatively cheap access to international markets in any part of the world through a highly integrated, efficient network of trunk routes and feeder services utilizing trans-shipment opportunities. Since the introduction of containerization, container throughput in the world has continuously increased due to economic growth, and several other factors including container penetration of general cargo trades, the movement of empty containers and increased trans-shipment. Owing to a combination of these factors, container throughputs have increased even in periods of regional recession, as was the case during the Asian economic crisis.

The sustained growth of container trade has been accompanied by the globalization of container shipping market. Severe competition among container shipping has forced owners to implement innovative, productivity-enhancing and cost-cutting strategies. Successively larger vessels have been employed on mainline trades. New service patterns have evolved, including ‘Round-the-World’ and ‘Pendulum’ services. In their search for cost reduction and faster transit times, lines have reduced the number of port calls, leading to the growth of ‘hubs’ or ‘load centres’ and the evolution of feeder networks. Very large (‘mega’) carriers are emerging and lines are entering into various types of strategic alliance. Currently, 4,000-6,000 TEU (twenty feet equivalent units) vessels already dominated major Asian deep-sea trades.

Since 2002, ships in excess of 6,500 TEU have come into operation on Asian routes and some carriers are considering constructing and deploying even larger ships. Increased concentration in liner shipping makes it vitally important for a port to keep its existing shipping company tenants. However, increased vessel size gives shipping lines incentives to look for ports with deeper access channels, berthing areas, and higher dockside and terminal efficiency to reduce the turn-around-time. It will also reduce the number of port calls to maximize the productivity of “mega vessel”. These factors not only force existing ports to invest in capacity improvement, but also provide opportunities for building new ports at potentially attractive sites. Therefore, modeling of container transportation network is important to predict the future liner shipping decision. Of course, the prediction is very useful for port authority to prepare the best strategy.

Some researchers have been concerned with container transportation network. Perakis and Jaramillo (1991) and Jaramillo and Perakis (1991) developed a linear programming model for a routing strategy to minimize total fleet operating and lay-up cost during a planning horizon. They assumed several predetermined routes (sequences of ports of call) and developed a model to assign each ship to some mix of the predetermined routes. Rana and Vickson (1991) presented nonlinear programming models.
They tried to maximize total profit by finding an optimal sequence of ports of call for each ship. Imai and Papadimitrious (1996), Osman, et al., (1997), and Ieda et al., (1998a, 1998b, 1999) focused on the carriers’ behavior of port choice and failed to model transportation tariffs. Also they did not consider the domestics shippers’ behavior or management policy of port administrators. In the actual market, three kinds of player exist: carriers; shippers, and port administrators. At least, market prediction must give equilibrium solution of behaviors of carriers and shippers under various port management policies. From this point of view, Kuroda et al (1999, 2000, 2001), Kuroda and Yang (1997a, 1997b), Yang (1999) developed different models of container marine transportation. Mauro, et al (2002) presented a successful application of integer linear programming to support the decisionmaking process of assigning ships. However, the above researchers did not consider the draft of vessels (vertical distance between the waterline and the keel). Why should port characteristics (berth depth) be considered on maritime transportation network model? Theoretically, 6000 TEU vessels will enter port with depth more than 15 m. However, in the real market, 6000 TEU ship can enter port with depth of 12m (e.g., Shanghai port). Why the phenomenon occurs? In this case, carriers do not use full capacity of 6000 TEU ship in order to ship can enter the port. Therefore, Concept of load factor ($\lambda$) of ship should be included on maritime transportation network model. In the previous model, load factor for each ship type is fixed. This paper try to accommodate concept of load factor.

The aim of this paper is to analyze container transportation network equilibrium considering draft of vessels. The rest of this paper is organized as follows. Section 2 describes model formulation. Section 3 provides numerical example. Finally, conclusion is given in section 4.

**METHOD**

In the real container market, three players should be considered, i.e. port administrator, ship companies (carriers), and shippers. Port administrators create strategy to take advantage of the maritime transportation market of the prosperity of their port by constructing of deep-water container berths, increasing port capacity, cargo handling charge, installing the electric data information system, etc. In accordance with port administrators’ strategy, Carriers also create strategy in order to survive in the market by choosing route, vessel type and service frequency on each route, taking into account the inter-port cargo volume. The strategy of carriers aims to minimize their total transportation cost. It should be noticed that the strategy of carriers is constrained by the policy of port administrator. In other side, shipper may choose their port and schedule choice to minimize the total transportation cost and loss of cargo value due to waiting at a port under a given inland transportation network and transportation service presented by carriers. Therefore, there is strong correlation among port administrator, carriers, and shippers. In the market context, this condition will leads to Nash equilibrium problem. Nash Equilibrium is a set of strategies such that player believe that it is doing the best it can, given the strategy of the other player (Damme, 1991).

The concept of the model is shown in Figure 1. In this study, a route that connect specific port (A, B, C, D) is called as “link” ($l$). Carriers can choose which port as loading port and unloading port; and they also can choose vessel type ($m$), service frequency ($f$), and container volume ($x$). Shippers can choose which link of land transportation ($l'$) and port to send a container from origin zone $i$ to destination zone $j$.  

**Assumptions**

The following assumptions are included in the study:

- The market is perfectly competitive.
- There are so many carriers and shippers in the market.
- Carriers can choose the loading and unloading port and they can’t choose marine transportation route.
- Carriers and shippers have perfect information about market.
- Carriers can choose some classes of vessel size.
- Navigation time on a specific link is same for different ship size. In the numerical example, 1000 TEU, 3000 TEU and 6000 TEU vessels are considered.
- The O.D cargo volume is a priori given and not influenced by services provided by carriers.
- Carriers have to transport all the O.D. cargo.
Carrier’s Behavior
As stated earlier, carriers create strategy in order to minimize total operation cost by choosing route network among port including service frequency and vessel size. Therefore, decision variables of carriers are, choosing service frequency of a vessel of type m on link l (f_l^n) and container volume transported on link l (x_l). Mathematically, it can be formulated as follows:

\[
\min Z_c = \sum_{l,m} C_i^n(f_l^n, x_l) \quad \text{[1]}
\]

\[
C_{i,m} = F_i^n(f_l^n) + \sum_h \delta_{ih} PC_i^n f_l^n + \sum_h \delta_{ih} x_l CW_h \quad \text{[2]}
\]

\[
F_i^n = [T_i^n(MFO_i^n + \gamma_{i,m}) f_l^n + \Phi(f_l^n)] \quad \text{[3]}
\]

\[
\Phi(f_l^n) = \theta_{l,m} \left( \frac{\sum_{l,m} \delta_{i,l} \lambda_{i,m} f_l^n CP_i^n}{VC_h} \right) \quad \text{[4]}
\]

Subject to:
\[
\sum_i \sum_j \sum_r \delta_{r,i} x_{j}^{r} = x_l \quad \text{for all l} \quad \text{[5]}
\]

\[
x_l \leq \sum_l \delta_{i,l} \lambda_{i,m} f_l^n CP_i^n \quad \text{for all l} \quad \text{[6]}
\]

\[
\sum_l \sum_i \delta_{i,l} \lambda_{i,m} f_l^n CP_i^n \leq VC_h \quad \text{[7]}
\]

\[
\sum_l \sum_i \delta_{i,l} \lambda_{i,m} f_l^n CP_i^n \leq VC_{i,m} \quad \text{for all h} \quad \text{[8]}
\]

\[
f_l^n \geq 0 \quad \text{for all l} \quad \text{[9]}
\]

Where:
- \(Z_c\) = total cost
- \(C_{i,m}\) = total operation cost of vessel of type m on link l
- \(f_l^n\) = service frequency of a vessel of type m on link l
- \(T_i^n\) = navigation time of a vessel of type m on link l
- \(F_i^n\) = total navigation cost of vessel type m on link l
- \(\delta_{i,l}^{\phi}\) = croneker’s delta, takes 1 when the port h is included on link l, otherwise takes zero
- \(\delta_{i,l}^{\phi L}\) = croneker’s delta, takes 1 when the port h is used as loading and unloading port on link l, otherwise takes zero
- \(MFO_i^n\) = fuel cost of a vessel of type m on link l
- \(CA_i^n\) = ship cost of a vessel of type m on link l
- \(PC_i^n\) = port charge of type m connected with link l
- \(x_l\) = container volume transported on link l
- \(CW_h\) = loading and unloading charge at port h
- \(\Phi(f_l^n)\) = congestion cost
- \(\theta_{l,m}\) = parameter
- \(CP_{i,m}\) = maximum loading capacity of vessel of type m on link l
- \(\lambda_{i,m}\) = load factor of a vessel of type m on link l
- \(VC_h\) = maximum capacity of port h

Function \(\Phi(f_l^n)\) in equation (3) means an additional cost coefficient due to congestion as increases of service frequency at port h expressed by equation (4). Eq. (5) means that container volume from zone i to zone j using link l of route r should equal to the total container flowing on link l. Eq. (6) represents link capacity constraint. Eq. (7) and Eq. (8) represent the port capacity constraints. Eq. (9) is a non-negative constraint for service frequency.

Shipper’s Behavior
Strategy of shippers is to choose the loading and unloading port in order to minimize total transport cost. Therefore, decision variable of shipper is to send container volume \(x_{i,j}^{w}\) from zone i to zone j by using port q as loading port (exporting port) and port s as unloading port (importing port). Mathematically, shipper’s behavior can be stated as follows:

\[
\min Z_s^{w} = SC_k \Gamma(x_{i,j}^{w}) \quad \text{[10]}
\]

\[
SC_k = \delta_{k} \left( \sum_{l} \delta_{i} (TV_{l} \times T_{l}) (1 + \rho(x_{i,j})) + \delta_{i} CP_{l} \right) + \delta_{i} CP_{l} \quad \text{[11]}
\]

\[
\xi(x_{i,j}) = \rho \left( \frac{x_{i,j}}{\sum_{l} \lambda_{l,m} CP_{l} f_l^n} \right) \quad \text{[12]}
\]

Subject to:
\[
\sum_i \sum_j \sum_{q,s} \delta_{i} x_{ij}^{w} = x_{ij} \quad \text{for all l} \quad \text{[13]}
\]

\[
\sum_i \sum_j \sum_{q,s} \delta_{i} x_{ij}^{w} = x_{ij} \quad \text{for all l} \quad \text{[14]}
\]

\[
\sum_i \sum_j \sum_{q,s} \delta_{i} x_{ij}^{w} = x_{ij} \quad \text{for all l} \quad \text{[15]}
\]

\[
x_{ij}^{w} \geq 0 \quad \text{for all i,j,q,s} \quad \text{[16]}
\]

Where:
- \(Z_s^{w}\) = shipper’s total cost between origin i and destination j
- \(SC_k\) = shipper’s total cost using route k
- \(X_{ij}\) = OD cargo volume from zone i to zone j
- \(\Gamma(x_{i,j})\) =
  \begin{cases} 
  1 & \text{when } x_{ij}^{w} > 0 \\
  0 & \text{otherwise}
  \end{cases}
  \]
- \(\delta_{i}^{w}\) =
  \begin{cases} 
  1 & \text{when route } r \text{ is included in route } k \\
  0 & \text{otherwise}
  \end{cases}
- \(\delta_{i}^{w}\) =
  \begin{cases} 
  1 & \text{when route } r' \text{ is included in route } k \\
  0 & \text{otherwise}
  \end{cases}
- \(\delta_{i}^{w}\) =
  \begin{cases} 
  1 & \text{when port } q \text{ and } s \text{ in included on link } l \\
  0 & \text{otherwise}
  \end{cases}
- \(\delta_{i}^{w}\) =
  \begin{cases} 
  1 & \text{when port } q \text{ and } s \text{ in included on link } l' \\
  0 & \text{otherwise}
  \end{cases}

\( TV \) = time value of cargo  
\( T_l \) = total travel time of link \( l \)  
\( P_r \) = marine transportation tariff of route \( r \)  
\( C_{l'} \) = inland transportation cost of link \( l' \)  
\( \xi(x_l) \) = link congestion cost function  
\( CP_l \) = total loading capacity of link \( l \)  
\( f_m^l \) = service frequency of vessel of type \( m \) on link \( l \)  
\( \rho_1, \rho_2 \) = parameter

In Eq. (11), function \( \xi(x_l) \) denotes the additional cost coefficient due to congestion at port connected with link \( l \) as express by Eq. (12). Eq. (13) is constraint of OD cargo volume. Eq. (14) and (15) refers to constraint for container volume on link on a link. The equilibrium tariff is given by:

\[ P_r = \sum \delta^l_i \frac{C_{l'}}{x_j} \quad \text{for} \quad \forall \ l \quad \text{.........................}[17] \]

RESULTS AND DISCUSSION

To examine the accuracy of the model, numerical example is carried out. Some major ports in the world which have significant container volume throughput are included. List of these ports is shown in Table 1.

Table 1. Port and hinterland

<table>
<thead>
<tr>
<th>No.</th>
<th>Port (Hinterland)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Keihin (Tokyo and Yokohama)</td>
</tr>
<tr>
<td>2</td>
<td>Nagoya</td>
</tr>
<tr>
<td>3</td>
<td>Hanshin (Osaka and Kobe)</td>
</tr>
<tr>
<td>4</td>
<td>Kitakyushu, Shimonoseki, Hakata</td>
</tr>
<tr>
<td>5</td>
<td>Busan</td>
</tr>
<tr>
<td>6</td>
<td>Kaohsiung</td>
</tr>
<tr>
<td>7</td>
<td>Shanghai</td>
</tr>
<tr>
<td>8</td>
<td>Hong Kong</td>
</tr>
<tr>
<td>9</td>
<td>Singapore</td>
</tr>
<tr>
<td>10</td>
<td>Tanjung Priok</td>
</tr>
<tr>
<td>11</td>
<td>Port Klang</td>
</tr>
<tr>
<td>12</td>
<td>Manila</td>
</tr>
<tr>
<td>13</td>
<td>Laem Chabang</td>
</tr>
<tr>
<td>14</td>
<td>Los Angeles</td>
</tr>
<tr>
<td>15</td>
<td>Rotterdam</td>
</tr>
</tbody>
</table>

Table 2. Number of vessels on each port

<table>
<thead>
<tr>
<th>Vessel type</th>
<th>1000 TEU</th>
<th>3000 TEU</th>
<th>6000 TEU</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Keihin</td>
<td>108</td>
<td>1920</td>
</tr>
<tr>
<td>2</td>
<td>Nagoya</td>
<td>140</td>
<td>286</td>
</tr>
<tr>
<td>3</td>
<td>Hanshin</td>
<td>131</td>
<td>1173</td>
</tr>
<tr>
<td>4</td>
<td>Kit, Shim, Hako</td>
<td>215</td>
<td>91</td>
</tr>
<tr>
<td>5</td>
<td>Busan</td>
<td>312</td>
<td>467</td>
</tr>
<tr>
<td>6</td>
<td>Kaohsiung</td>
<td>40</td>
<td>1039</td>
</tr>
<tr>
<td>7</td>
<td>Shanghai</td>
<td>763</td>
<td>781</td>
</tr>
<tr>
<td>8</td>
<td>Hong Kong</td>
<td>1377</td>
<td>1229</td>
</tr>
<tr>
<td>9</td>
<td>Singapore</td>
<td>398</td>
<td>274</td>
</tr>
<tr>
<td>10</td>
<td>Tanjung Priok</td>
<td>388</td>
<td>388</td>
</tr>
<tr>
<td>11</td>
<td>Port Klang</td>
<td>464</td>
<td>328</td>
</tr>
<tr>
<td>12</td>
<td>Manila</td>
<td>106</td>
<td>75</td>
</tr>
<tr>
<td>13</td>
<td>Laem Chabang</td>
<td>380</td>
<td>369</td>
</tr>
<tr>
<td>14</td>
<td>Los Angeles</td>
<td>1554</td>
<td>964</td>
</tr>
<tr>
<td>15</td>
<td>Rotterdam</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: * indicates port with depth less than 15 m

To validate the model we compare container transshipment on specific port which has transshipment data as shown in Table 4 and Figure 2. Figure 2 shows that coefficient of determination \( R^2 \) is 0.95. The result indicates the model provide good accuracy.
Table 3. Container throughput (TEU)

<table>
<thead>
<tr>
<th>No.</th>
<th>Port Name</th>
<th>Export (TEU)</th>
<th>Import (TEU)</th>
<th>Tranship (TEU)</th>
<th>Pass (TEU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Keihin</td>
<td>1,575,789</td>
<td>2,657,409</td>
<td>1,154,876</td>
<td>2,117,466</td>
</tr>
<tr>
<td>2</td>
<td>Nagoya</td>
<td>867,595</td>
<td>723,626</td>
<td>423,968</td>
<td>487,902</td>
</tr>
<tr>
<td>3</td>
<td>Hanshin</td>
<td>816,469</td>
<td>1,355,698</td>
<td>673,984</td>
<td>1,493,749</td>
</tr>
<tr>
<td>4</td>
<td>Kit, Shim, Hak</td>
<td>382,092</td>
<td>559,601</td>
<td>178,936</td>
<td>401,256</td>
</tr>
<tr>
<td>5</td>
<td>Busan</td>
<td>2,194,084</td>
<td>2,036,346</td>
<td>1,196,279</td>
<td>3,151,354</td>
</tr>
<tr>
<td>6</td>
<td>Kaohsiung</td>
<td>2,241,737</td>
<td>1,862,187</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Shanghai</td>
<td>3,027,020</td>
<td>2,324,898</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Hong Kong</td>
<td>6,143,814</td>
<td>5,772,629</td>
<td>5,592,450</td>
<td>6,753,984</td>
</tr>
<tr>
<td>9</td>
<td>Singapore</td>
<td>1,984,562</td>
<td>2,045,773</td>
<td>8,765,924</td>
<td>8,074,254</td>
</tr>
<tr>
<td>10</td>
<td>Tanjung Priok</td>
<td>1,011,060</td>
<td>1,184,955</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>Port Klang</td>
<td>1,871,988</td>
<td>1,748,795</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>Manila</td>
<td>582,765</td>
<td>665,925</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>Laem Chabang</td>
<td>1,298,913</td>
<td>1,181,587</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>Los Angeles</td>
<td>6,682,194</td>
<td>7,899,196</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>Rotterdam</td>
<td>5,653,951</td>
<td>4,315,408</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4. Comparison of container transshipment between model and data

<table>
<thead>
<tr>
<th>No.</th>
<th>Port Name</th>
<th>Transhipment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>1</td>
<td>Keihin</td>
<td>1,154,876</td>
</tr>
<tr>
<td>2</td>
<td>Nagoya</td>
<td>423,968</td>
</tr>
<tr>
<td>3</td>
<td>Hanshin</td>
<td>673,984</td>
</tr>
<tr>
<td>4</td>
<td>Kit, Shim, Hak</td>
<td>178,936</td>
</tr>
<tr>
<td>5</td>
<td>Busan</td>
<td>1,198,279</td>
</tr>
<tr>
<td>6</td>
<td>Hong Kong</td>
<td>5,592,450</td>
</tr>
<tr>
<td>7</td>
<td>Singapore</td>
<td>8,765,924</td>
</tr>
</tbody>
</table>

Figure 2. Comparison between data and model of Transshipment

CONCLUSION
This paper analyzes container transportation network equilibrium considering draft of vessels. Concept of load factor (λ) of ship is included in the model. In the real container market, three players should be considered, i.e. port administrator, ship companies (carriers), and shippers. Port administrators create strategy to take advantage of the maritime transportation market of the prosperity of their port. In accordance with port administrators' strategy, Carriers also create strategy in order to survive in the market by choosing route, vessel type and service frequency on each route. In other side, shipper may choose their port and schedule choice to minimize the total transportation cost and loss of cargo value due to waiting at a port under a given inland transportation network and transportation service presented by carriers. In the market context, this condition will leads to Nash equilibrium problem. The result of the model calculation indicates that Hong Kong and Singapore port dominates container throughput in the world and the big vessel (3000 - 6000 TEU) is dominant in these ports. Conversely, the smaller port with depth less than 15 m dominated by 1000 TEU vessels. The result is inline with the reality. Currently, the two biggest ports previously mentioned are the hub port and serve the smaller port. The other finding from the study is 6000 TEU vessels can enter port with depth less than 15 m such as port of Shanghai. Again, this result is inline with reality. If we did not consider the concept of load factor, of course the result will provide different result which differs from reality. Validation of the model shows that coefficient of determination (R²) is 0.95. The result indicates the model provides good accuracy.

REFERENCES:


